Two-Loop QCD Renormalization and Anomalous Dimension of the Scalar Diquark Operator

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Abstract

The renormalization of the scalar diquark operator and its anomalous dimension is calculated at two-loop order in QCD, enabling higher-order QCD studies of diquarks. As an application of our result, the two-loop diquark anomalous dimension in the $\overline{\text{MS}}$ scheme is used to study the QCD renormalization scale dependence of diquark matrix elements of the $\Delta S = 1$ effective weak Hamiltonian.

1 Introduction

Four-quark (or tetraquark) $qq\bar{q}\bar{q}$ states explain the inverted mass hierarchy of the scalar mesons compared to a $q\bar{q}$ nonet in a variety of theoretical approaches [1, 2, 3, 4, 5]. With the inclusion of a gluonium (glueball) state [6], the scalar spectrum below 2 GeV is then understood as mixtures of gluonium, the $q\bar{q}$ nonet, and the $qq\bar{q}\bar{q}$ nonet. The $X(3872)$ [7] and $Y(4260)$ [8] mesons can also be interpreted as four-quark states [9].

Diquark (qq) clusters are relevant to the internal structure of hadrons (see e.g., [10, 11]). In particular, Ref. [9] uses constituent models for diquark clusters to study four-quark states. The constituent (scalar) diquark masses that emerge in Ref. [9] are in good agreement with QCD sum-rule analyses of diquarks [12, 13], providing QCD corroboration for the diquark model of four-quark states.

In this paper, we study the renormalization of scalar diquark operators to two-loop order in QCD and thereby obtain the two-loop anomalous dimension of the scalar diquark current. As discussed below, the renormalization of the diquark operator is an essential component of QCD sum-rule analyses, and the anomalous dimension is also necessary for determining the scale dependence of matrix elements of the effective weak Hamiltonian for non-leptonic strange particle decays [14]. Our two-loop results thus enable future QCD studies of diquarks to higher loop order.

The scalar diquark operator in an anti-triplet colour configuration (the "good" diquark in the terminology of Ref. [11]) is given by [12]

$$J_\gamma = \epsilon_{\alpha\beta\gamma} Q_i^\alpha \left(C\gamma_5\right)_{ij} q_j^\beta = \epsilon_{\alpha\beta\gamma} Q_\alpha^T C\gamma_5 q_\beta \,, \tag{1}$$

where the greek and latin indices respectively represent colour and spin degrees of freedom for the quark fields Q and q, and C is the charge conjugation operator. The presence of a transposed quark field in (1) implies that the Feynman rule for the three-point function of the diquark operator and $\bar{Q}$, $\bar{q}$ fields shown in Fig. 1

$$\Gamma_d^{(0)} = -\epsilon_{\alpha\beta\gamma} C\gamma_5, \tag{2}$$

implicitly transposes the external propagator associated with the Q field.

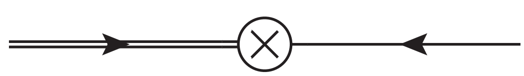

Figure 1: Feynman diagram for the tree-level vertex of the diquark operator with the quark fields $\bar{Q}$ and $\bar{q}$. The double line represents the Q field that is transposed and the diquark operator is denoted by $\otimes$. This and all subsequent Feynman diagrams were drawn with JaxoDraw [15].

2 One-Loop Renormalization

Although the diquark operator is gauge dependent, the theory of composite-operator renormalization [16] implies that the diquark operator is multiplicatively renormalizable because there are no lower-dimension operators with the same quantum numbers as (1).[1] The one-loop renormalization of the diquark operator can thus determined by Fig. 2, which results in the following one-particle irreducible (1PI) Green function for a zero-momentum insertion of J_γ in D-dimensions (dimensional regularization)

$$\Gamma_d^{(1)} = i\frac{g^2}{4}\lambda^a_{\sigma\alpha}\lambda^a_{\tau\beta}\epsilon_{\sigma\tau\gamma}\frac{1}{\nu^{2\epsilon}}\int\frac{d^Dk}{(2\pi)^D}\left(\gamma^\rho\right)^T\frac{(\not p+\not k)^T}{(p+k)^2}C\gamma_5\frac{(\not p+\not k)}{(p+k)^2}\gamma^\mu\left[-\frac{g_{\mu\rho}}{k^2}+(1-\xi)\frac{k_\mu k_\rho}{k^4}\right], \tag{3}$$

where ν is the renormalization scale, the quark mass has been ignored because dimensional regularization is a mass-independent scheme, $\alpha_s = g^2/(4\pi)$, colour indices have been explicitly shown for the Gell-Mann matrices λ^a, and a covariant gauge with gauge parameter ξ has been used. Working in normal (or naive) dimensional regularization,[2] where $\{\gamma^\mu,\gamma_5\}=0$ [18] in $D=4+2\epsilon$ dimensions, and using the (D-dimensional) properties of the charge conjugation operator $CC=-1$ and $C\left(\gamma_\mu\right)^T C=\gamma_\mu$ [19] we find

$$\Gamma_d^{(1)} = \frac{8}{3}\left[-\epsilon_{\alpha\beta\gamma}C\gamma_5\right]i\frac{g^2}{4}\frac{1}{\nu^{2\epsilon}}\int\frac{d^Dk}{(2\pi)^D}\gamma^\rho\frac{(\not p+\not k)}{(p+k)^2}\frac{(\not p+\not k)}{(p+k)^2}\gamma^\mu\left[-\frac{g_{\mu\rho}}{k^2}+(1-\xi)\frac{k_\mu k_\rho}{k^4}\right]. \tag{4}$$

By comparison with the one-loop process determining the renormalization of the scalar current $J_s=\bar{Q}q$, we see that (4) can be related to the (one-loop) 1PI result for the scalar current $\Gamma_s^{(1)}$ apart from a numerical factor C_d representing the ratio of the different colour factors that occur in the two processes

$$\Gamma_d^{(1)} = \frac{1}{2}\Gamma_d^{(0)}\Gamma_s^{(1)} \equiv C_d\Gamma_d^{(0)}\Gamma_s^{(1)}, \tag{5}$$

as represented diagrammatically in Fig. 3.

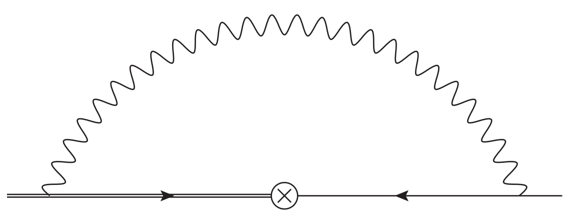

Figure 2: One-loop Feynman diagram for the renormalization of J_γ. As in Fig. 1, the double line represents the (transposed) Q field and the diquark operator is denoted by $\otimes$.

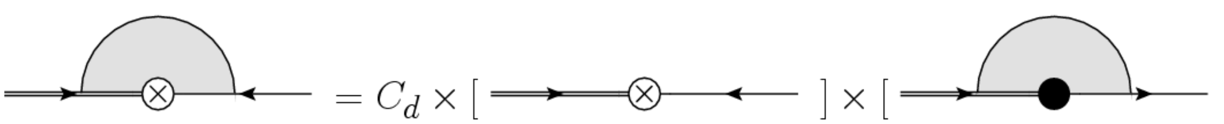

Figure 3: Diagrammatic representation of the relationship (5) between two-point functions with scalar and diquark operator insertions. The scalar operator is denoted by the solid circle.

The renormalized diquark operator $[J_\gamma]_R$ is defined via the renormalization constant Z_d,

$$[J_\gamma]_R = Z_d J_\gamma\,. \tag{6}$$

Similarly, the well-known renormalization of the scalar operator is

$$[J_s]_R = Z_m J_s \tag{7}$$

[1] We are grateful for discussions with John Dixon clarifying this point.

[2] We have chosen to work in normal dimensional regularization (as opposed to, e.g., the 't Hooft-Veltman scheme [17]) because QCD sum-rule analyses of diquarks [12, 14] have used the normal dimensional regularization scheme.

where Z_m is the quark mass renormalization constant. Using (5) it is easy to see that to one-loop order in the minimal-subtraction (MS) and associated schemes

$$Z_d = Z_{2F}^{1/2} Z_m^{1/2}\,, \tag{8}$$

where Z_{2F} is the renormalization constant for the quark fields. Landau gauge ($\xi = 0$) is of particular interest in the QCD sum-rule analysis of diquark currents, because the Schwinger string used for a gauge-invariant formulation of the two-point diquark correlation function vanishes in this gauge [12]. Combining the one-loop Landau-gauge result $Z_{2F} = 1$ with (8) leads to the one-loop Landau gauge MS-scheme result

$$Z_d = Z_m^{1/2} = 1 + \frac{1}{2}\frac{\alpha}{\pi}\frac{1}{\epsilon}\,, \tag{9}$$

where we use the dimensional regularization convention $D = 4+2\epsilon$. Eq. (9) agrees with the (one-loop) renormalization and renormalization-group improvement implicitly implemented in Refs. [12, 14].

3 Two-Loop Renormalization

The two-loop diagrams for the renormalization of the diquark operator are shown in Fig. 4. As in the one-loop analysis and shown in Fig. 3, each diagram is given by a colour factor C_d multiplying the bare diquark vertex and the equivalent diagram with a scalar current. The divergent parts for each of the two-loop diagrams in Fig. 4 are expressed in Table 1 in terms of the corresponding scalar diagram $\Gamma^{(2)}_{s,i}$ in the modified minimal-subtraction ($\overline{\rm MS}$) scheme

$$\Gamma^{(2)}_{s,i} = \left(\frac{\alpha_b}{\pi}\right)^2\left[\frac{A_i}{\epsilon} + \frac{B_i}{\epsilon^2}\right]\,, \quad i \in \{1\,,2\,,\ldots\,11\}\,, \tag{10}$$

where n_f is the number of active quark flavours and α_b and ξ_b are the bare coupling and gauge parameter. A number of the Feynman diagrams are clearly related by the exchange of Q and q fields, and hence Table 1 exhibits anticipated symmetries $\Gamma_4 = \Gamma_6$, $\Gamma_7 = \Gamma_8$ and $\Gamma_9 = \Gamma_{11}$. Note that the colour factors C_d that relate the scalar and diquark diagrams are not universally equal to the one-loop result $C_d = 1/2$, implying that one cannot expect the simple pattern of the one-loop result (9) to persist at two-loop order. The diagrams that are the exception to the one-loop pattern (Γ_5 and Γ_{10}) require multiple applications of colour algebra identities unique to the Feynman rule (2); all other diagrams contain a single application of these identities combined with standard colour algebra factors occuring in the renormalization of the scalar operator.[3]

The two-loop renormalization procedure first involves the replacement of α_b and ξ_b with their (one-loop) renormalized expressions (see, e.g., Ref. [21])

$$Z_\alpha = 1 + \frac{\alpha}{\pi}\left[\frac{33-2n_f}{12\epsilon}\right]\,, \quad \alpha_b = Z_\alpha \alpha\,; \tag{11}$$

$$Z_\xi = 1 + \frac{\alpha}{\pi}\left[\frac{4n_f - 39 + 9\xi}{24\epsilon}\right]\,, \quad \xi_b = Z_\xi \xi\,. \tag{12}$$

in the two-loop 1PI Green function

$$\Gamma_d = \Gamma^{(0)}_d + \Gamma^{(1)}_d + \Gamma^{(2)}_d\,. \tag{13}$$

For consistency at two-loop level, (13) requires inclusion of the finite parts of the one-loop calculation (5)

$$\Gamma^{(1)}_s = \frac{1}{3}\left(\frac{\alpha_b}{\pi}\right)\left[-\frac{3+\xi_b}{\epsilon} + 2\,(2+\xi_b) - L\,(3+\xi_b)\right]\,, \quad L = \log\left[-\frac{p^2}{\nu^2}\right]. \tag{14}$$

The renormalization constant Z_d is then constrained by the requirement that it cancel the divergences in

$$Z_d Z_{2F}\left[\Gamma^{(0)}_d + \Gamma^{(1)}_d + \Gamma^{(2)}_d\right]\,, \tag{15}$$

where the two-loop $\overline{\rm MS}$ quark field renormalization constant is [22]

$$Z_{2F} = 1 + \frac{\alpha}{\pi}\frac{\xi}{3\epsilon} + \left(\frac{\alpha}{\pi}\right)^2\left[\frac{\xi\,(27+17\xi)}{144\epsilon^2} + \frac{201 - 12n_f + 72\xi + 9\xi^2}{288\epsilon}\right]\,. \tag{16}$$

[3] In the previous version of this paper the Table 1 colour factor for diagram 10 in Fig. 4 was erroneous [20].

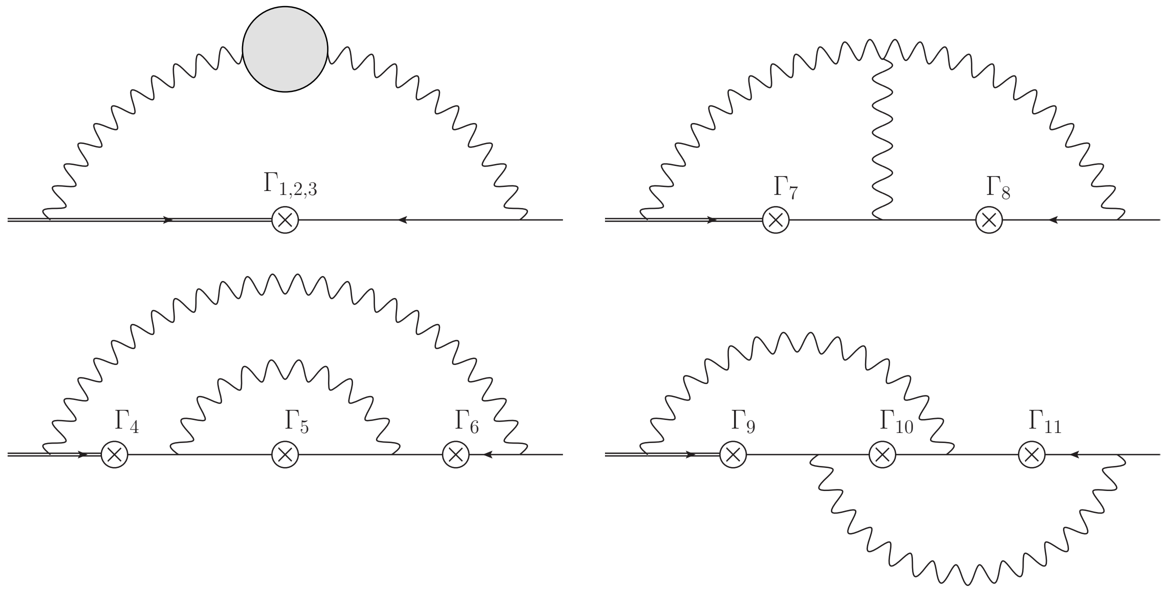

Figure 4: Two-loop diagrams for the renormalization of the diquark operator where Γ_1 denotes a quark loop, Γ_2 a ghost loop and Γ_3 a gluon loop. Implicitly, the Q (double) line extends to the insertion of the diquark operator.

As a benchmark to ensure accuracy in our calculations in Table 1, we have verified that our results for the scalar diagrams lead to the required two-loop $\overline{\rm MS}$ result $Z_s = Z_m$ [23]

$$Z_m = 1 + \frac{\alpha}{\pi\epsilon} + \left(\frac{\alpha}{\pi}\right)^2 \left[\frac{1}{\epsilon^2}\left(\frac{15}{8} - \frac{n_f}{12}\right) + \frac{1}{\epsilon}\left(\frac{101}{48} - \frac{5n_f}{72}\right)\right] . \tag{17}$$

The final QCD result for the two-loop $\overline{\rm MS}$ diquark renormalization constant is

$$Z_d = 1 + \frac{\alpha}{\pi}\left[\frac{3-\xi}{6\epsilon}\right] + \left(\frac{\alpha}{\pi}\right)^2 \left[\frac{1}{\epsilon}\left(\frac{1545 - 40n_f}{2880} - \frac{\xi}{8} - \frac{\xi^2}{64}\right) + \frac{1}{\epsilon^2}\left(\frac{234 - 12n_f}{288} - \frac{17\xi}{96} - \frac{5\xi^2}{288}\right)\right] . \tag{18}$$

The cancellation of the L/ϵ terms in Z_d that are generated by (14) provides another consistency check on our calculation. Note that the two-loop Landau gauge result does not uphold the one-loop ($\xi = 0$) pattern $Z_d = Z_m^{1/2}$.

The anomalous dimension for the diquark operator defined by

$$\gamma_d = \frac{\nu}{Z_d}\frac{dZ_d}{d\nu} , \tag{19}$$

is easily extracted from (18) to obtain the two-loop $\overline{\rm MS}$ QCD anomalous dimension for the diquark operator

$$\gamma_d(\alpha) = \gamma_1 \frac{\alpha}{\pi} + \gamma_2 \left(\frac{\alpha}{\pi}\right)^2 , \tag{20}$$

$$\gamma_1 = 1 - \frac{\xi}{3} , \; \gamma_2 = \frac{1545 - 40n_f}{720} - \frac{\xi}{2} - \frac{\xi^2}{16} . \tag{21}$$

In the extraction of the anomalous dimension we have verified that the two-loop coefficients of Z_d

$$Z_d = 1 + \frac{Z_{d,1}}{\epsilon} + \frac{Z_{d,2}}{\epsilon^2} + \ldots \tag{22}$$

satisfy the renormalization-group constraint

$$2\alpha\frac{\partial Z_{d,2}}{\partial\alpha} = \left[\gamma_d(\alpha) - \beta(\alpha)\alpha\frac{\partial}{\partial\alpha} - \delta(\alpha,\xi)\xi\frac{\partial}{\partial\xi}\right] Z_{d,1} , \tag{23}$$

i	C_d	A_i	B_i
1	$\frac{1}{2}$	$\frac{n_f(2-L)}{6}$	$-\frac{n_f}{12}$
2	$\frac{1}{2}$	$\frac{(2L-5)\left(1+\xi_b^2\right)}{32}$	$\frac{1+\xi_b^2}{32}$
3	$\frac{1}{2}$	$\frac{\xi_b^2+4\xi_b-44-L\left(\xi_b^2+6\xi_b-25\right)}{16}$	$\frac{25-6\xi_b-\xi_b^2}{32}$
4	$\frac{1}{2}$	$\frac{\xi_b[5+2\xi_b-L(3+\xi_b)]}{9}$	$-\frac{\xi_b(3+\xi_b)}{18}$
5	$\frac{1}{4}$	$\frac{(3+\xi_b)[2L(3+\xi_b)-11-5\xi_b]}{18}$	$\frac{(3+\xi_b)^2}{18}$
6	$\frac{1}{2}$	$\frac{\xi_b[5+2\xi_b-L(3+\xi_b)]}{9}$	$-\frac{\xi_b(3+\xi_b)}{18}$
7	$\frac{1}{2}$	$\frac{3L\left(\xi_b^2+4\xi_b+3\right)-5\xi_b^2-17\xi_b-24}{16}$	$\frac{3\left(\xi_b^2+4\xi_b+3\right)}{32}$
8	$\frac{1}{2}$	$\frac{3L\left(\xi_b^2+4\xi_b+3\right)-5\xi_b^2-17\xi_b-24}{16}$	$\frac{3\left(\xi_b^2+4\xi_b+3\right)}{32}$
9	$\frac{1}{2}$	$-\frac{(3+\xi_b)[1+\xi_b(L-2)]}{72}$	$-\frac{\xi_b(3+\xi_b)}{144}$
10	$\frac{5}{2}$	$\frac{3-6\xi_b-\xi_b^2}{144}$	0
11	$\frac{1}{2}$	$-\frac{(3+\xi_b)[1+\xi_b(L-2)]}{72}$	$-\frac{\xi_b(3+\xi_b)}{144}$

Table 1: Results for the two-loop diagrams in Fig. 4. The quantity $L = \log(-p^2/\nu^2)$ and the notations for A_i and B_i are defined in Eq. (10).

where we are working in the conventions of [21] with the (one-loop) β function and anomalous dimension δ of the gauge parameter given by

$$\beta(\alpha) = \beta_1 \frac{\alpha}{\pi}\,,\ \beta_1 = -\frac{11}{2} + \frac{n_f}{3} \tag{24}$$

$$\delta(\alpha,\xi) = \delta_1 \frac{\alpha}{\pi}\,,\ \delta_1 = \frac{1}{4}\,(13-3\xi) - \frac{n_f}{3}\,. \tag{25}$$

Confirmation of this renormalization-group constraint provides another verification of the accuracy of our results given in Table 1.

4 Application and Conclusions

It has previously been noted that at leading-order, the renormalization scale dependence cancels between the QCD perturbative contributions to the diquark decay constants and the $\Delta S = 1$ effective weak Hamiltonian, although there remains some residual scale dependence from non-perturbative terms [14]. As an application of our two-loop results, we can explore this scale dependence at next-to-leading order. Following Ref. [14], we consider the combination

$$c_-(\mu) g_+(\mu) g_+(\mu) \tag{26}$$

where $c_-(\mu)$ represents the renormalization scale dependence of the Wilson coefficient in the $\Delta S = 1$ effective weak Hamiltonian [24] and $g_+(\mu)$ is the scale-dependent scalar diquark decay constant emerging from QCD sum-rules [14]. The renormalization-group (RG) factor arising from c_- is [24]

$$c_-(\mu) \sim \exp\left[-\int \frac{\gamma_-(\alpha)}{\beta(\alpha)}\frac{d\alpha}{\alpha}\right], \tag{27}$$

where in the normal dimensional regularization scheme with $n_f = 3$, the anomalous dimension $\gamma_-(\alpha)$ is[4]

$$\gamma_-(\alpha) = \tilde{\gamma}_1 \frac{\alpha}{\pi} + \tilde{\gamma}_2 \left(\frac{\alpha}{\pi}\right)^2 \tag{28}$$

$$\tilde{\gamma}_1 = -2\,,\ \tilde{\gamma}_2 = -\frac{50}{48}\,. \tag{29}$$

Similarly, the anomalous dimension for the diquark operator leads to the following RG factor for the (scalar) diquark decay constants

$$g_+(\mu) g_+(\mu) \sim \exp\left[-2\int \frac{\gamma_d(\alpha)}{\beta(\alpha)}\frac{d\alpha}{\alpha}\right]. \tag{30}$$

As mentioned above, QCD sum-rule calculations with diquark currents extract gauge-invariant information from the two-point correlation function through the insertion of a Schwinger string, which becomes trivial for a line geometry in Landau gauge [12]. Thus for applications to RG behaviour of the diquark decay constants, we use (21) with $n_f = 3$ and $\xi = 0$:

$$\gamma_1 = 1,\ \gamma_2 = \frac{95}{48}\,. \tag{31}$$

The resulting RG behaviour of (26) is

$$c_-(\mu) g_+(\mu) g_+(\mu) \sim \exp\left[\int \frac{4}{9}\frac{\left[1+\frac{\tilde{\gamma}_2}{\tilde{\gamma}_1}\frac{\alpha}{\pi}\right]}{\left[1+\frac{\beta_2}{\beta_1}\frac{\alpha}{\pi}\right]}\frac{d\alpha}{\alpha}\right] \exp\left[-\int \frac{4}{9}\frac{\left[1+\frac{\gamma_2}{\gamma_1}\frac{\alpha}{\pi}\right]}{\left[1+\frac{\beta_2}{\beta_1}\frac{\alpha}{\pi}\right]}\frac{d\alpha}{\alpha}\right] = 1 - \frac{35}{54}\frac{\alpha(\mu)}{\pi}\ . \tag{32}$$

Thus the leading-order cancellation of scale dependence in (26) for the perturbative contributions to g_+ does not persist to second order. However, the residual scale dependence associated with (32), which decreases with increasing $\alpha(\mu)$, does have the right qualitative behaviour to counter the residual scale dependence encountered in Ref. [14]. A more detailed analysis of the residual scale dependence is beyond the scope of this paper because it would require a full next-order sum-rule analysis of the diquark decay constants.

In conclusion, we have determined the $\overline{\rm MS}$ renormalization constant and associated anomalous dimension for the scalar diquark operator at two-loop order in QCD in an arbitrary covariant gauge for normal dimensional regularization. This result enables future QCD sum-rule studies of diquarks to higher-orders in perturbation theory. For example, the divergent terms in the diquark renormalization constant (18) combined with lower-loop $\mathcal{O}(\epsilon)$ and $\mathcal{O}\left(\epsilon^2\right)$ terms generate finite parts corresponding to renormalization-induced physical contributions to the diquark correlation function. Furthermore, the anomalous dimension of the diquark operator appearing in the renormalization-group equation governing scale dependence of the diquark correlation function is an essential feature of QCD Laplace sum-rule analyses [25]. Given the relative size of the one- and two-loop terms in (18) and (31), these renormalization-induced and anomalous dimension effects could be significant in higher-loop extensions of [14].

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